Statistics

Lecture 12

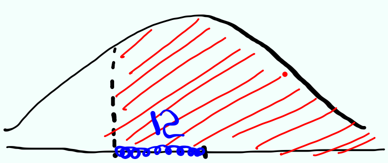
Feb 19-8:47 AM

Class Quiz 7

Drawing, labeling, shading, and full TI Command required.

Consider a normal prob. dist. with the mean of 86 and standard dev. of 12. $N(86, 12)$

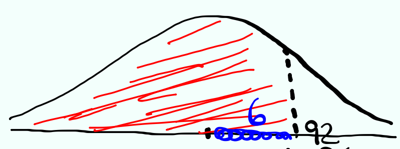
1) Find $P(x>74)$



$\mu=86$
$\sigma=12$ ✓

$= \text{normalcdf}(74, E99, 86, 12)$
$= \boxed{.841}$

2) Find $P(\bar{x}<92)$ for group of 4.



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 86 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{12}{\sqrt{4}} = 6 \end{cases}$ ✓

$= \text{normalcdf}(-E99, 92, 86, 6)$
$= \boxed{.841}$

Nov 15-7:20 AM

Estimating Parameters:

SG 22 & SG 23

Samples ← Statistic

Population ← Parameter

To estimate Parameter → we use similar statistic.

To estimate Pop. Proportion p — we use Sample Proportion $\hat{p}$ (P-hat)

Pop. Mean μ — Sample Mean $\bar{x}$

Point-estimate for p is $\hat{p}$.

" " " μ is $\bar{x}$.

Point-estimate

Nov 15-8:16 AM

When estimating a parameter, the answer is a $\underbrace{\text{range of values}}_{\text{Confidence interval}}$.

Every Conf. interval comes with Confidence level. (C-level)

Middle Area

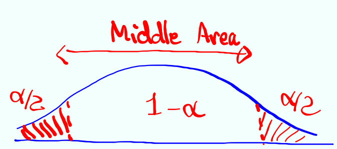
Middle Region

$(1-\alpha)\cdot 100\%$

Alpha

$0<\alpha<1$

Significance level



$\alpha/2$ is the area of each tail.

When C-level is not given → Use 95%

When α is not given → Use .05

Nov 15-8:21 AM

Confidence Interval for Population Proportion P:

$$\hat{p} - E < P < \hat{p} + E$$

Sample Proportion

Point-estimate

Margin of error

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Critical Value for $(1-\alpha)\cdot 100\%$ C-level.

$\hat{p} = \dfrac{x}{n}$ ← # of favorable responses / ← Sample Size

$\hat{q} = 1 - \hat{p}$

Nov 15-8:29 AM

I randomly surveyed 100 students and 80 of them voted for presidential election. Find $\boxed{98\% \text{ conf. interval}}$ for the prop. of all students that voted in Presidential election.

C-level: .98

$n = 100$

$x = 80$

$\hat{p} = \frac{x}{n} = \frac{80}{100} = .8$

$\hat{q} = 1 - \hat{p} = .2$

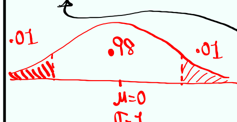
$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 2.326 \cdot \sqrt{\frac{(.8)(.2)}{100}} \approx \boxed{.09}$

$\hat{p} - E < P < \hat{p} + E$

$.8 - .09 < P < .8 + .09$

$\boxed{.71 < P < .89}$



$Z_{\alpha/2} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

We are 98% Confident that between 71% & 89% of all College students voted in Presidential election.

Using TI:

[STAT] [TESTS] [1-PropZInt]

$x = 80$

$n = 100$

C-level: .98

[Calculate]

$.706 < P < .893$

$\boxed{.71 < P < .89}$

$E = \frac{.89 - .71}{2} = .09$

$\hat{p} = \frac{.89 + .71}{2} = .8$

Nov 15-8:34 AM

I surveyed 250 students and 40 were smokers. Find $\boxed{90\%\text{ Conf. interval}}$ for the prop. of all students that are smokers.

C-level: .9

$n = 250$

$x = 40$

$\hat{p} = \frac{x}{n} = \frac{40}{250} = .16$

$\hat{q} = 1 - \hat{p} = .84$

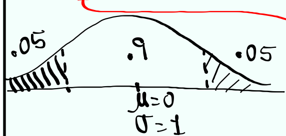
$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 1.645 \cdot \sqrt{\frac{(.16)(.84)}{250}} \approx \boxed{.04}$

$\hat{p} - E < p < \hat{p} + E$

$.16 - .04 < p < .16 + .04$

$\boxed{.12 < p < .20}$



$z_{\alpha/2}$: invNorm$(.95, 0, 1) \approx \boxed{1.645}$

STAT
TESTS
1-PropZInt
x=40
n=250
C-level: .9
Calculate
.12 < P < .20

$E = \frac{.2 - .12}{2} = .04$

$\hat{p} = \frac{.2 + .12}{2} = .16$

Nov 15-8:48 AM

I surveyed 240 registered voters and 72% of them trusted the outcome of Presidential election.

$n = 240$

$\hat{p} = .72$

$\hat{p} = \frac{x}{n}$

$x = n\hat{p} = 240(.72) = 172.8 \rightarrow \boxed{x = 173}$

if decimal → Round-up

Find $\boxed{99\%\text{ Confidence}}$ interval for the prop. of all registered voters that trusted the outcome of election.

→ C-level: .99

1-PropZInt
x=173
n=240
C-level: .99
Calculate

$.65 < p < .80$

$E = \frac{.80 - .65}{2} = .075 \approx 7.5\%$

$\hat{p} = \frac{.80 + .65}{2} = .725 \approx 72.5\%$

Nov 15-8:59 AM

Confidence Interval for Population mean μ:

$$\bar{x} - E < \mu < \bar{x} + E$$

Sample Mean

Point-estimate

Margin of error

Case I: σ Known

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

TI:

$\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$ $\boxed{\text{Z Interval}}$

inpt: $\boxed{\text{Stats}}$

Nov 15-9:19 AM

A sample of 25 students had a mean age of 30 yrs. $\boxed{n=25}$ $\boxed{\bar{x}=30}$

C-level: .9

Find $\boxed{90\% \text{ Conf. interval}}$ for mean age of all students if $\sigma = 8$.

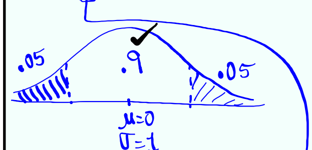
$$\bar{x} - E < \mu < \bar{x} + E$$

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$30 - 2.6 < \mu < 30 + 2.6$$

$$= 1.645 \cdot \frac{8}{\sqrt{25}} \approx \boxed{2.6}$$

$$27.4 < \mu < 32.6$$



$$\boxed{27 < \mu < 33}$$

$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

$\boxed{\text{STAT}}$

$\boxed{\text{TESTS}}$

$\boxed{\text{Z Interval}}$

inpt: $\boxed{\text{Stats}}$

$\sigma = 8$

$\bar{x} = 30$

$n = 25$

C-level: .9

$\boxed{\text{Calculate}}$

$27 < \mu < 33$

$$E = \frac{33 - 27}{2} = 3$$

$$\bar{x} = \frac{33 + 27}{2} = 30$$

Nov 15-9:24 AM

Salaries of $\underset{=}{\text{All}}$ nurses are normally dist. with standard dev. of $400. $\sigma = 400$

A sample of 30 nurses had a mean monthly salary of $7500. $n = 30$ $\bar{x} = 7500$

No C-level

Find $\boxed{\text{Conf. interval}}$ for the mean salary of all nurses. Use .95

σ Known $\Rightarrow$ Z Interval

inpt: $\boxed{\text{Stats}}$

$\sigma : 400$

$\bar{x} = 7500$

$n = 30$

C-level: .95

$\boxed{\text{Calculate}}$

$\boxed{7357 < \mu < 7643}$

$E = \dfrac{7643 - 7357}{2} = \boxed{143}$

$\bar{x} = \dfrac{7643 + 7357}{2} = \boxed{7500}$

Nov 15-9:33 AM

Confidence Interval for Population mean μ:

$$\bar{x} - E < \mu < \bar{x} + E$$

Sample Mean

Point-estimate

Margin of error

Case I: σ Known	Case II: σ unknown
$E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \dfrac{s}{\sqrt{n}}$ $df = n - 1$
TI: $\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$ $\boxed{\text{Z Interval}}$ inpt: $\boxed{\text{Stats}}$	TI: $\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$ $\boxed{\text{T Interval}}$ inpt: $\boxed{\text{Stats}}$

Nov 15-9:19 AM

Given: $n = 15$, $\bar{x} = 32.5$, $S = 6.5$

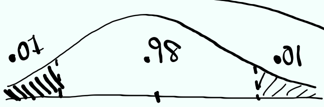
C-level: .98 $\quad$ **σ unknown**

Find Conf. interval for pop. mean μ.

$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$ $\qquad$ $\bar{x} - E < \mu < \bar{x} + E$

$= 2.624 \cdot \frac{6.5}{\sqrt{15}} = \boxed{4.4}$ $\qquad$ $32.5 - 4.4 < \mu < 32.5 + 4.4$

$\boxed{28.1 < \mu < 36.9}$



$\mu = 0$

σ unknown

$df = n - 1 = 14$

$t_{\alpha/2} = \text{invT}(.99, 14) \approx \boxed{2.624}$

T Interval

inpt: $\boxed{\text{Stats}}$

$\bar{x} = 32.5$

$S = 6.5$

$n = 15$

C-level: .98

$\boxed{\text{Calculate}}$

Nov 15-9:44 AM

A Sample of 12 teachers from LAUSD had a mean Salary of \$7750 with Standard deviation of \$450.

$n = 12$

$\bar{x} = 7750$

$S = 450$

σ Unknown ⇒ T Interval

Find 99% Conf. interval for the mean Salary of all teachers.

T Interval

inpt: $\boxed{\text{Stats}}$

$\bar{x} = 7750$ (whole)

$S = 450$

$n = 12 \rightarrow df = n - 1 = 11$

C-level: .99

$\boxed{7347 < \mu < 8154}$

$E = \frac{8154 - 7347}{2} \approx 404$

$\bar{x} = \frac{8154 + 7347}{2} \approx 7751$

Nov 15-9:53 AM

I randomly Selected 12 exams, here are the Scores

75	82	68	90
95	100	70	88
80	65	100	58

Store in L1,
Use $\boxed{\text{1 - Var stats}}$

$\bar{x} = 80.9$
$S = 14.0$
Round to 1-dec.

NO C-level → Use .95

Find $\boxed{\text{Conf. interval}}$ for the mean of all exams. σ unknown ⇒ T Interval

inpt: $\boxed{\text{Stats}}$

$\bar{x} = 80.9$
$S = 14$
$n = 12$
C-level: .95
$\boxed{\text{Calculate}}$

$72.0 < \mu < 89.8$ one decimal

$$E = \frac{89.8 - 72.0}{2} = \boxed{8.9}$$

$$\bar{x} = \frac{89.8 + 72.0}{2} = \boxed{80.9}$$

Nov 15-10:00 AM

How to determine minimum $\underbrace{\text{Sample Size}}_{n}$:

Proportion

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

with Some algebra work

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

if decimal → Round-up

If $\hat{p}$ & $\hat{q}$ are unknown, then use .5 for each

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

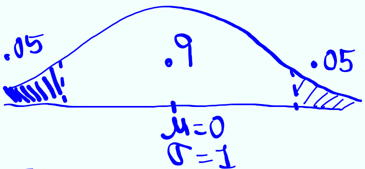
Nov 15-10:19 AM

Find minimum Sample Size needed to Construct 90% Conf. interval for pop. prop. with margin of error not to exceed 6%.

1) Assume $\hat{p} = .4$

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= (.4)(.6)\left(\frac{1.645}{.06}\right)^2$$



$n \approx 180.401\ldots$

$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

$\boxed{n \approx 181}$

2) Assume $\hat{p}$ & $\hat{q}$ are both unknown

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.645}{.06}\right)^2 \approx 187.918$$

$\boxed{n \approx 188}$

Nov 15-10:23 AM

Find minimum $\overset{n}{\text{Sample Size}}$ needed to Construct $\boxed{\text{Conf. interval}}$ for pop. prop. and error not exceed 8%.

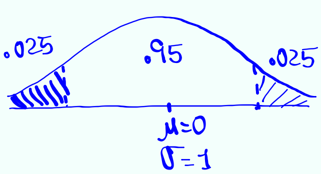
→ No C-level → use .95

→ $E = .08$

1) Assume $\hat{p} = .25$

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= (.25)(.75)\left(\frac{1.960}{.08}\right)^2$$



$= 112.547$

$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

$\boxed{n \approx 113}$

2) Assume $\hat{p}$ & $\hat{q}$ are both unknown

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.960}{.08}\right)^2 = 150.0625$$

$\boxed{n \approx 151}$

Nov 15-10:31 AM

How to determine minimum $\underbrace{\text{Sample Size}}_{n}$:

Population Mean

$E = Z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$ with some algebra work

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

If decimal $\Rightarrow$ Round-up

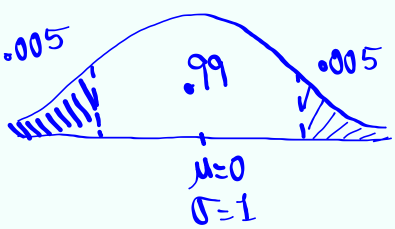
When σ unknown $\rightarrow$ Use S instead.

$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E}\right)^2$$

Nov 15-10:19 AM

Find minimum Sample Size needed to Construct 99% Conf. interval for pop. mean with $\sigma = 25$ & $E = 10$.

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$



$$= \left(\frac{2.576 \cdot 25}{10}\right)^2$$

$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

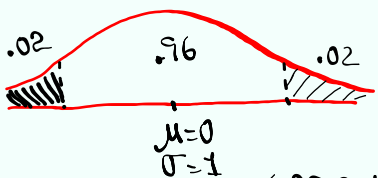
$= 41.474$

$\boxed{n \approx 42}$

Nov 15-10:43 AM

Find minimum Sample Size needed to Construct 96% Conf. interval for pop. mean and error not to exceed 8 and $S = 12.5$.

σ unknown $\quad n = \left(\dfrac{Z_{\alpha/2} \cdot S}{E}\right)^2$



$Z_{\alpha/2} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

$n = \left(\dfrac{2.054 \cdot 12.5}{8}\right)^2 = 10.3\ldots$

$\boxed{n \approx 11}$

Redo with $E = 5$

$n = \left(\dfrac{2.054 \cdot 12.5}{E}\right)^2 = 26.368 \quad \boxed{n \approx 27}$

Nov 15-10:48 AM

Given $\quad n = 15, \quad \bar{x} = 28, \quad S = 8$

1) Find $\boxed{\text{Conf. interval}}$ for Pop. mean.

NO C-level

σ unknown $\rightarrow$ TInterval

C-level : .95 $\qquad \boxed{24 < \mu < 32}$

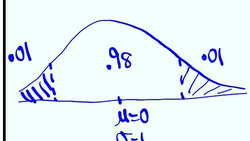
2) Find E.

$= \dfrac{32 - 24}{2} = 4$

3) Find minimum Sample Size needed to Construct 98% Conf. interval for pop. mean and margin of error not to exceed 2.5.

$n = \left(\dfrac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$

use S



$Z_{\alpha/2} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

$= \left(\dfrac{2.326 \cdot 8}{2.5}\right)^2$

$\boxed{n \approx 56}$

Use $E = 5$

$n = \left(\dfrac{2.326 \cdot 8}{5}\right)^2 \quad \boxed{n \approx 14}$

SG 22 & 23 ✓

Nov 15-10:55 AM